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The LNM Institute of Information Technology

The LNM Institute of Information Technology
Department of Physics
M. Sc. Admission Entrance Exam, 2018

Date: 1st August 2018 Time: 10.30AM -12.00 PM Total Marks: 40

Sample Questions with Solutions

Instructions:

1. There are 8 questions carrying equal marks. Attempt all the questions.
2. Tick mark the correct answer with **ball pen** only and not with pencil.
3. Show your derivation in the answer book. There will be partial marking for your attempt.
4. Use of calculator is allowed.

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1. If $\vec{A} = A\hat{i}$ is a constant vector with one non-zero component and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is the position vector, then $\vec{\nabla} \cdot (\vec{r} \times \vec{A})$ is equal to

   (a) ✓ 0 (b) A (c) $\vec{A}$ (d) $\vec{r}$

   Solution: Here the components of the vector $\vec{A} = A\hat{i}$. Now,

$$\begin{aligned}\vec{r} \times \vec{A} &= yA\hat{j} \times \hat{i} + zA\hat{k} \times \hat{i} \\ &= -yA\hat{k} + zA\hat{j} \\ \vec{\nabla} &= \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z} \\ \Rightarrow \vec{\nabla} \cdot (\vec{r} \times \vec{A}) &= 0\end{aligned}$$

2. A system of ideal gas goes through a process defined by $p = p_0 e^{-V/V_0}$. The maximum attainable temperature by the system is

   (a) $\frac{p_0}{enk_B V_0}$ (b) $\frac{p_0}{nk_B V_0}$ (c) ✓ $\frac{p_0 V}{enk_B}$ (d) $\frac{nk_B}{p_0}$

   Hint: Along with the given relation, use the equation of state for ideal gas ($pV = nk_BT$) to find a relation between the temperature and volume or pressure.

   Solution: As suggested in the hint,

$$p = nk_BT/V \text{ as the gas is "Ideal".} \tag{1}$$

It undergoes a process defined by,

$$p = p_0 e^{-V/V_0}. \tag{2}$$

Thus during the process its temperature and volume is related by,

$$T = \frac{p_0}{nk_B} V e^{-V/V_0}. \tag{3}$$

Now

$$\frac{dT}{dV} = \frac{p_0}{nk_B}\left(e^{-V/V_0} - \frac{V}{V_0}e^{-V/V_0}\right)$$
$$\frac{d^2T}{dV^2} = \frac{p_0}{nk_B}\left(-\frac{2}{V_0}e^{-V/V_0} + \frac{V}{V_0^2}e^{-V/V_0}\right)$$

For extremum:

$$\frac{dT}{dV} = 0$$
$$\Rightarrow V = V_0 \tag{4}$$

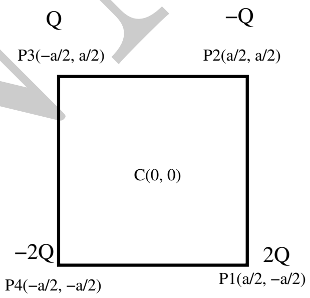
At $V = V_0$:

$$\frac{d^2T}{dV^2} = -\frac{p_0}{nk_B V_0 e} < 0 \tag{5}$$

i.e $V = V_0 = V_{max}$ corresponds to maxima and thus the maximum attainable temperature is

$$T_{max} = \frac{p_0 V_0}{nk_B e}. \tag{6}$$

3. Four point charges of amount $\pm Q$ and $\pm 2Q$ are placed at the four corners $P1, P2, P3$ and $P4$ of a square (in x-y plane) of length a as follows.



The electric field generated by them at the centre C with coordinate (0,0) is directed along:

(a) ✓ $\hat{i}$ (b) $\hat{j}$ (c) $\hat{i} + \hat{j}$ (d) None of the above

Solution: The contribution from the charge at P_1 is given as

$$\vec{E_1} = \frac{\text{Amount of charge at P}_1}{\textit{Square of the distance between } P_1 \textit{ and the center}} \textit{Unit vector towards } P_1 \textit{ to the center}$$
$$= \frac{2Q}{a^2/2}\frac{1}{\sqrt{2}}(-\hat{i} + \hat{j})$$
$$= \frac{Q}{\sqrt{2}a^2}4(-\hat{i} + \hat{j}) \tag{7}$$

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$$\vec{E}_2 = \frac{-2Q}{a^2}\frac{1}{\sqrt{2}}(-\hat{i}-\hat{j}) \\ = \frac{Q}{\sqrt{2}a^2}2(\hat{i}+\hat{j}) \tag{8}$$

$$\vec{E}_3 = \frac{2Q}{a^2}\frac{1}{\sqrt{2}}(\hat{i}-\hat{j}) \\ = \frac{Q}{\sqrt{2}a^2}2(\hat{i}-\hat{j}) \tag{9}$$

$$\vec{E}_4 = \frac{-4Q}{a^2}\frac{1}{\sqrt{2}}(\hat{i}+\hat{j}) \\ = \frac{Q}{\sqrt{2}a^2}4(-\hat{i}-\hat{j}) \tag{10}$$

$$\vec{E}_1+\vec{E}_2+\vec{E}_3+\vec{E}_4 = \frac{Q}{\sqrt{2}a^2}(-4\hat{i}+0\times\hat{j}) = \frac{4Q}{\sqrt{2}a^2}\hat{i} \tag{11}$$

4. The energy E of a one dimensional system as a function of its position x can be written as $E(x) = -a/x^2 + b/x^4$, where a and b are positive constants. The equilibrium position of the system is given as:

   (a) ✓ $x = \pm\sqrt{2b/a}$ (b) $x = \pm\sqrt{a/2b}$

   (c) $x = \pm\sqrt{a/b}$ (d) None of the above

   Solution:

$$\frac{dE(x)}{dx} = +2a/x^3 - 4b/x^5 \\ \frac{d^2E(x)}{dx^2} = -6a/x^4 + 20b/x^6 \tag{12}$$

$$\frac{dE(x)}{dx} = 0 \\ \Rightarrow x = \pm\sqrt{2b/a} = x_0 \text{ (say)} \tag{13}$$

   At $x = x_0$

$$\frac{d^2E(x)}{dx^2} = -3a^3/2b^2 + 5a^3/2b^2 = a^3/b^2 > 0 \tag{14}$$

   Thus equilibrium positions are located at $x_0 = \pm\sqrt{a/2b}$.

5. If a simple cubic unit cell is maximally occupied (closed packing) by spherical atoms placed at its corners. Then the fraction of the space occupied by the atoms in an unit cell is given as:

   (a) ✓ $\frac{\pi}{6}$ (b) $\frac{\pi}{5}$

   (c) $\frac{\pi}{3}$ (d) None of the above.

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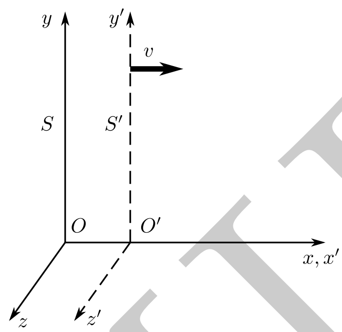
Solution: Let a be the edge length of the unit cell and r be the radius of spherical atoms. As spheres are touching each other, therefore $a = 2r$. No. of spheres per unit cell $= 1/8 \times 8 = 1$. Volume of the sphere $= \frac{4}{3}\pi r^3$. Volume of the cube $= a^3 = (2r)^3 = 8r^3$. Thus the fraction of the space occupied $= \frac{4}{3}\pi r^3/8r^3 = \frac{\pi}{6} = 0.524 = 52.4\%$

6. For an observer O, two events (A and B) occur simultaneously 600 km apart. Another observer O' measures the distance between the same two events to be 1200 km. Then the the velocity of the observer O' (in terms of the velocity of light c) with respect to the observer O is



(a) ✓ $\frac{\sqrt{3}}{2}c$

(b) $\frac{3}{2}c$

(c) $\frac{\sqrt{2}}{3}c$

(d) None of the above.

Here c is the velocity of light.

Note: We use the standard inertial frames S (laboratory frame) and S' (rest frame of any event) which are set up such that the x and x' axes coincide and the y, y' axes and z, z' axes are parallel. Seen from S, S' moves in the positive x-direction with speed v. Clocks in both frames are set to zero when the origins O and O' coincide.

Solution: We know,

$$x'_A = \frac{x_A - vt_A}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{15}$$

and

$$x'_B = \frac{x_B - vt_B}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{16}$$

Thus,

$$x'_B - x'_A = \frac{x_B - x_A - v(t_B - t_A)}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{17}$$

Now $x_B - x_A = 6 \times 10^5 m$, $x'_B - x'_A = 12 \times 10^5 m$ and $t_B - t_A = 0$. Thus

$$12 \times 10^5 = \frac{6 \times 10^5 m - v \times 0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{18}$$

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Or

$$
\begin{aligned}
\sqrt{1-\frac{v^2}{c^2}} &= \frac{1}{2} \\
\Rightarrow 1-\frac{v^2}{c^2} &= \frac{1}{4} \\
\Rightarrow \frac{v}{c} &= \frac{\sqrt{3}}{2}
\end{aligned} \tag{19}
$$

7. A particle is confined in an one dimensional box defined by the coordinates $(-L/2, L/2)$. If $\psi_n(x) = \frac{2}{\sqrt{L}} \cos\frac{n\pi x}{L}$ is the normalised eigen function of the particle for the n-th energy state, its momentum expectation value in its n-the energy eigen state is given as:

   (a) ✓ 0 (b) $n\hbar/L$ (c) $\hbar/nL$ (d) $0.5\hbar/L$

Solution:

$$
\begin{aligned}
< p >_n &= \int_{-L/2}^{L/2} dx \psi_n^*(x) \left(-i\hbar\frac{\partial}{\partial x}\right)\psi_n(x) \\
&= -i\hbar\frac{n\pi}{L}\int_{-L/2}^{L/2} \cos\frac{n\pi x}{L}\sin\frac{n\pi x}{L} dx
\end{aligned} \tag{20}
$$

$$
\begin{aligned}
I &= \int_{-L/2}^{L/2} \cos\frac{n\pi x}{L}\sin\frac{n\pi x}{L} dx \\
&= \int_{-L/2}^{0} \cos\frac{n\pi x}{L}\sin\frac{n\pi x}{L} dx + \int_{0}^{L/2} \cos\frac{n\pi x}{L}\sin\frac{n\pi x}{L} dx \\
&= \int_{L/2}^{0} \cos\frac{n\pi x}{L}\sin\frac{n\pi x}{L} dx + \int_{0}^{L/2} \cos\frac{n\pi x}{L}\sin\frac{n\pi x}{L} dx \text{ (With x} \to -\text{x in the first term).} \\
&= 0
\end{aligned} \tag{21}
$$

8. Two light rays of amplitudes a and $a/2$ interfere at a point where they have a phase difference ϕ. Then the resultant intensity I_{tot} is **[5]**

   (a) ✓ $I_{tot} = a^2(\frac{5}{4} + \cos\phi)$ (b) $I_{tot} = a^2(\frac{5}{4} - \cos\phi)$

   (c) $I_{tot} = a^2(\frac{5}{4} + \sin\phi)$ (d) $I_{tot} = a^2(\frac{5}{4} - \sin\phi)$

Solution: Total intensity

$$
I_{tot} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi \tag{22}
$$

Here $I_1 = a^2$, $I_2 = a^2/4$, thus

$$
\begin{aligned}
I_{tot} &= \frac{5}{4}a^2 + 2\sqrt{a^4/4}\cos\phi \\
&= a^2(\frac{5}{4} + \cos\phi)
\end{aligned} \tag{23}
$$

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