

The LNM Institute of Information Technology, Jaipur
Entrance Test for M.Sc. (Mathematics) 2016

Name:

Application ID:

Father's Name:

Date:

Each question carries 4 marks. Tick mark (\surd) the correct answer. Only one option is correct. Time limit is 60 minutes. **There is a negative marking.** One mark will be deducted for each incorrect answer.

1. The order and degree of the differential equation $2x \frac{d^2y}{dx^2} + 5x^2 \left\{ \left(\frac{dy}{dx} \right)^4 - xy^2 \right\}^{\frac{1}{2}} = 0$ are

- (a) Second order & first degree.
- (b) First order & fourth degree.
- (c) Second order & fourth degree.
- (d) Second order & second degree.

2. Which of the following is **correct**.

- (a) A vector space may have more than one zero vector.
- (b) If f and g are polynomials of degree n , then $f + g$ is a polynomial of degree n .
- (c) Subsets of linearly dependent sets are linearly dependent.
- (d) None of these

3. Let

$$f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

Then

- (a) $\lim_{x \rightarrow 0} f(x)$ exist.
- (b) $\lim_{x \rightarrow 0} f(x)$ does not exist.
- (c) f is discontinuous everywhere.
- (d) None of these.

4. Let

$$f(x, y) = \begin{cases} \frac{-3xy}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

Then

- (a) f is continuous at $(0, 0)$ but is not differentiable at $(0, 0)$.
- (b) Both $f_x(0, 0)$ and $f_y(0, 0)$ exist but f is not differentiable at $(0, 0)$.
- (c) f is continuous at $(0, 0)$ and $f_x(0, 0)$ and $f_y(0, 0)$ exist
- (d) None of these.

5. How many fields are there (up to isomorphism) with exactly 6 elements?

- (a) 0
- (b) 1
- (c) 5
- (d) 6

6. Which one is **not correct** ?

- (a) The curl of any gradient is the zero vector.
- (b) The divergence of any curl is zero.
- (c) Line integral of gradient vector field along a closed curve is zero.
- (d) None of these.

7. Let

$$P = \{(x, y) \in \mathbb{R}^2 \mid 0 < x^2 < 1\}$$

$$Q = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x^2 + y^2 < 1\}$$

$$R = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$$

Then

- (a) All P, Q, R are open subsets of \mathbb{R}^2 .
- (b) Both P and Q are open and R is not open.
- (c) Only P is open
- (d) None of these.

8. The initial value problem $\frac{dy}{dx} = \frac{y-1}{x}$, $y(0) = 1$ has

- (a) Unique solution
- (b) Exactly two solutions
- (c) Infinitely many solutions
- (d) No solution

9. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be the linear transformation defined by

$$T(a_1, a_2) = (a_1 + 3a_2, 0, 2a_1 - 4a_2).$$

Then which of the following matrix represents T ?

- (a) $\begin{bmatrix} 1 & 3 \\ 0 & 0 \\ 2 & -4 \end{bmatrix}$
- (b) $\begin{bmatrix} 1 & 0 & 2 \\ 3 & 0 & -4 \end{bmatrix}$
- (c) $\begin{bmatrix} 1 & 3 \\ -3 & 0 \\ 2 & -4 \end{bmatrix}$
- (d) $\begin{bmatrix} 2 & 3 \\ 2 & -1 \end{bmatrix}$

10. For $0 < \theta < \pi$, the matrix $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$

- (a) is skew symmetric
- (b) is not orthogonal
- (c) has no real eigenvalues
- (d) None of these

11. Which of the following matrix is (are) not diagonalizable over \mathbb{R} :

- (a) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
- (b) $\begin{pmatrix} 0 & -3 \\ 2 & 5 \end{pmatrix}$
- (c) $\begin{pmatrix} 1 & 2 \\ -1 & 4 \end{pmatrix}$
- (d) $\begin{pmatrix} 1 & -5 \\ 0 & 1 \end{pmatrix}$

12. Let W be a subspace of a vector space V and S be a linearly dependent set in W . Which of the following statements is **FALSE**?

- (a) If S_1 is a subset of V such that $S_1 \supseteq S$, then S_1 must be linearly dependent.
- (b) If $\alpha \in V$ and $\alpha \notin W$, then $S \cup \{\alpha\}$ is linearly dependent.
- (c) There exists a subset $S_1 \subset S$ such that S_1 is linearly independent.
- (d) Every nonempty subset of S is linearly dependent

13. If the function $f(x) = \frac{ax+b}{x^2-1}$ has a local extreme value of 1 at $x = 3$ then

- (a) $a = 0, b = 1$
- (b) $a = 1, b = 0$
- (c) $a = b = 1$
- (d) None of these.

14. Which of the following pair of functions is not a linearly independent pair of solutions of $y'' + 9y = 0$

- (a) $\sin 3x, \sin 3x - \cos 3x$
- (b) $\sin 3x, \sin 3x \cos 3x$
- (c) $\sin 3x + \cos 3x, 3 \sin x - 4 \sin^3 x$
- (d) $\sin 3x + \cos 3x, 4 \cos^3 x - 3 \cos x$

15. Suppose C is the boundary of the set $\{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, 0 \leq y \leq 1\}$. Let
- $$\alpha = \int_C xy^2 dx + (x^2y + 2x)dy.$$
- be evaluated in counter-clockwise direction. Then
- (a) $\alpha = 1$
 (b) $\alpha = -1$
 (c) $\alpha = 2$
 (d) $\alpha = -2$
16. The maximum magnitude of the directional derivative for the surface $x^2 + xy + yz = 9$ at the point $(1, 2, 3)$ is along the direction
- (a) $\mathbf{i} + \mathbf{j} + \mathbf{k}$
 (b) $2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$
 (c) $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$
 (d) $\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$
17. Let $\{x_n\}$ and $\{y_n\}$ be any two real sequences. Then which of the following statement is correct ?
- (a) If $\{x_n y_n\}$ is convergent, and if $\{y_n\}$ converges, then $\{x_n\}$ is convergent.
 (b) If $\left\{\frac{x_n}{y_n}\right\}$ is convergent, then both $\{x_n\}$ and $\{y_n\}$ are convergent.
 (c) If $\{x_n\}$ is convergent and $\{y_n\}$ is divergent, then $\{x_n y_n\}$ is divergent.
 (d) None of these.
18. Which of the following statement is **not** correct ?
- (a) If an element of a group has order 1 then it is identity.
 (b) In the additive group $(\mathbb{R}, +)$, $\sqrt{2}$ has infinite order .
 (c) In the multiplicative group $(\mathbb{Q} - \{0\}, \times)$ all nonidentity elements have infinite order.
 (d) In the additive group $(\mathbb{Z}_9, +)$ the element 6 has order 3.
19. Which of the following are permissible orders for subgroups of a group of order 120?
- (a) 1, 2, 5, 7.
 (b) 1, 2, 5, 9.
 (c) 1, 2, 5, 15.
 (d) 2, 5, 60, 240.
20. Which of the following statement is correct ?
- (a) Every finite cyclic group is isomorphic to the additive group of $(\mathbb{Z}, +)$.
 (b) A cyclic group can be non-abelian.
 (c) A group of prime order is necessarily cyclic.
 (d) None of these.

