The LNM Institute of Information Technology, Jaipur

Entrance Test for M.Sc. (Mathematics) 2017

Name: Application ID:

Father's Name: Date:

Each question carries 4 marks. Tick mark ($\surd$) the correct answer. Only one option is correct. Time limit is 60 minutes. **There is a negative marking**. One mark will be deducted for each incorrect answer.

1. Which of the following sets are groups under multiplication modulo 11?
   (a) A = {1, 3, 5, 7, 8}.
   (b) B = {1, 8}.
   (c) A = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
   (d) C ={1,10}.

2. Which of the following group is cyclic ?.
   (a) Klein-4-group.
   (b) D_4.
   (c) The group of roots (real or complex) of the equation $x^n - 1 = 0$.
   (d) The group Q^* of non-zero rationals, for multiplication.

3. If p is a prime number and G is a non-abelian group of order p^3. The order of its centre is?
   (a) 0.
   (b) p.
   (c) p^2.
   (d) p^3

4. Let
$$f(x,y) = \begin{cases} \dfrac{x^3+2y^3}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$
   Which of the following is correct ?
   (a) f is continuous at $(0,0)$ but is not differentiable at $(0,0)$.
   (b) Both $f_x(0,0)$ and $f_y(0,0)$ exist but f is not differentiable at $(0,0)$.
   (c) f is continuous at $(0,0)$ and $f_x(0,0)$ and $f_y(0,0)$ exist
   (d) All the above.

5. If the surfaces $ax^2 - byz = (a+2)x$ and $4x^2y + z^3 = 4$ cut orthognally at $(1,-1,2)$. Then the values of a and b are
   (a) $\frac{5}{2}, 1$.
   (b) $\frac{3}{2}, 1$.
   (c) $\frac{5}{2}, 0$.
   (d) $\frac{3}{2}, 0$

6. If **a** is a constant vector and $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Then $Curl(\mathbf{a} \times \mathbf{r})$ is
   (a) **a**.
   (b) **2a**.
   (c) **3a**.
   (d) **0**.

7. If $f(x) = x^3 + px = q$ for $x \in \mathbb{R}$, where $p, q \in \mathbb{R}$ and $p > 0$, then f has
   (a) One real root.
   (b) Exactly two real roots.
   (c) Three real roots.
   (d) None of the above.

8. If A is a 4×6 matrix, what is the smallest possible dimension of null space of A?
   (a) 2
   (b) 4
   (c) 6
   (d) None of the above.

9. Let $V(F)$ denotes the vector space over the field F, then which of the following is infinite dimension space, where C is the complex field, R is the real field and Q is the rational filed.
   (a) C(R)
   (b) R(R)
   (c) R(Q)
   (d) Q(Q)

10. If a set of linear equations is represented by the matrix equation $Ax = b$, then the necessary condition for the existence of a solution for this system is:
   (a) A must be invertible
   (b) A must be singular
   (c) b must be linearly independent of the columns of A
   (d) b must be linearly depended on the columns of A

11. If $M = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$, then the eigenvalues of $M + 4I$ are
   (a) 1 & 3
   (b) 5 & 3
   (c) 5 & 7
   (d) None of above

12. Suppose $f : \mathbb{R} \longrightarrow \mathbb{R}$ is continuous and $f(x) = 1$ for all $x \in \mathbb{R} \setminus \mathbb{Q}$. Then
   (a) $f(0) = 0$
   (b) $f(0) = 1$
   (c) $f(0) \neq 0$
   (d) $f(0)$ may not be 1.

13. The closure of $A = \{\frac{1}{n} : n \in \mathbb{N}\}$ in $\mathbb{R}$ is
   (a) A
   (b) $\emptyset$
   (c) $[0, 1]$
   (d) none of the above.

14. Suppose (x_n) be a sequence with infinite distinct terms in a discrete metric space X, then
   (a) (x_n) is Cauchy but not convergent
   (b) (x_n) is convergent
   (c) (x_n) can't be a Cauchy sequence
   (d) none of the above.

15. Which of the following subspace of the real line is compact?
   (a) $\mathbb{Q}$
   (b) $(0, 1)$
   (c) $\{\{\frac{1}{n} : n \in \mathbb{N}\}$
   (d) $\{\frac{1}{n} : n \in \mathbb{N}\} \cup \{0\}$

16. Let U and V be two open subsets of a connected metric space X such that $W = U \cap V$ is compact. Then
   (a) $W = \emptyset$
   (b) $W \neq X$
   (c) $W = X$
   (d) $W \neq \emptyset$.

17. Which of the following subspace of the real line $\mathbb{R}$ is connected?
   (a) $[1, 2] \cap \mathbb{Q}$.
   (b) $\mathbb{Q}$
   (c) $\{\frac{1}{n} : n \in \mathbb{N}\}$
   (d) None of the above.

18. The interval of convergence of $\sum_{n=0}^{\infty} \frac{x^n}{\sqrt{n^2+3}}$ is
   (a) $[-1, 1]$
   (b) $(-1, 1)$
   (c) $[-1, 1)$
   (d) $[-1, 1)$.

19. Consider the initial value problem $\frac{dy}{dx} = \sqrt{|y|}$; $y(0) = 0$. Then this IVP has/have
   (a) unique solution
   (b) infinitely many solutions
   (c) no solution
   (d) only two solutions.

20. Let y_1 and y_2 be any two solution of the homogeneous differential equation $y'' + (x^2 + 2)y' + y = 0$ on the interval $[-1, 1]$. Furthermore $y_1; y_2$ have a common zero x_0 in this interval. Then y_1 and y_2

    (a) must be linearly independent

    (b) must be linearly dependent

    (c) may be linearly independent

    (d) none of the above

21. If one solution of $xy'' - (2x + 1)y' + (x + 1)y = 0$ is $y_1 = e^x$, then second solution y_2 is given by

    (a) xe^x

    (b) e^x

    (c) $\frac{e^x}{2}$

    (d) $\frac{x^2}{2}e^x$

Answer Key for Exam A

1. Which of the following sets are groups under multiplication modulo 11?

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   (b) $\frac{3}{2}, 1$.

   (c) $\frac{5}{2}, 0$.

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   (d) $\mathbf{0}$.

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